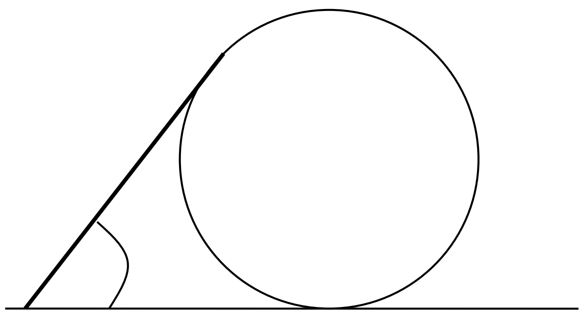
Teacher notes
Topic A

A problem on torques

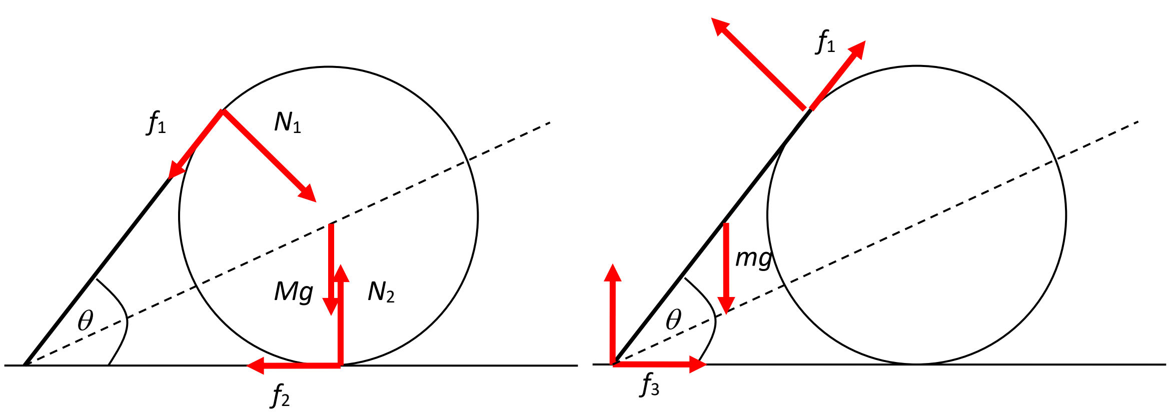
A rod of mass *m* and length *L* making an angle θ to the horizontal rests on a ring of radius *R* as shown. The rod is tangent to the ring. The ring is at rest.



Draw the forces on the ring and the rod.

Hence find the frictional force on the ring.

We have the obvious normal forces N_1, N_2 and the weight Mg of the ring. The rod pushes the ring to the right so frictional forces f_1 and f_2 develop as shown.



Taking torques about the center of the ring gives $f_1R = f_2R$ and so we learn that $f_1 = f_2 = f$.

Balancing horizontal forces: $N_1 \sin\theta = f + f\cos\theta$. (1)

Balancing vertical forces: $N_1 \cos\theta + Mg = N_2$. (2)

Now consider the forces *on the rod*.

Taking torques about the point where the rod touches the ground: $mg \times \frac{L}{2}\cos\theta = N_1 L$. (3)

So (3) gives: $N_1 = \frac{mg\cos\theta}{2}$ and so from (1):

$$f = \frac{N_1 \sin\theta}{1 + \cos\theta} = \frac{mg\cos\theta\sin\theta}{2(1 + \cos\theta)}$$